DIFFERENTIATING POLYNOMIALS | KEY FACTS

$$y = ax^n \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$$

MULTIPLY BY THE ORIGINAL POWER.

REDUCE THE POWER BY 1

There are two types of notation you will encounter in the A-level course. You need to be familiar with the following:

- Leibniz's Notation:
  - $y = 3x^2 \quad \frac{\mathrm{d}y}{\mathrm{d}x} = 6x$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2\right) = 6x$
- Lagrange's Notation:
  - $\mathrm{f}(x) = 3x^2 \quad \mathrm{f}'(x) = 6x$

DIFFERENTIATING POLYNOMIALS | KEY FACTS

- $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}(x) \pm \mathrm{g}(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(x) \pm \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{g}(x)$

  TERMS CAN BE DIFFERENTIATED SEPARATELY
  - e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 + 5x\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2\right) + \frac{\mathrm{d}}{\mathrm{d}x}(5x) = 6x + 5$
- $\frac{\mathrm{d}}{\mathrm{d}x}\left(k\,\mathrm{f}(x)\right) = k\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(x)$
  - e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2\right) = 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) = 3 \cdot 2x = 6x$

INCREASING AND DECREASING FUNCTIONS

- When the graph is increasing the gradient is positive.
  - $\mathrm{f}'(x) > 0$
- When the graph is decreasing the gradient is negative.
  - $\mathrm{f}'(x) < 0$

CORRECT FORM FOR DIFFERENTIATION | EXAMPLE PROBLEM PAIRS

1E. (a) $y = (x+2)(x-5)$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

EXPAND THE BRACKETS

$$y = (x+2)(x-5)$$
$$y = x^2 - 3x + 10$$
$$\frac{dy}{dx} = 2x - 3$$

(b) $\mathrm{f}(x) = x^2\sqrt{x}$. Find $\mathrm{f}'(x)$.

WRITE ANY SURDS USING INDEX NOTATION

$$f(x) = x^2\sqrt{x}$$
$$= x^2 \cdot x^{\frac{1}{2}}$$
$$= x^{\frac{5}{2}}$$
$$f'(x) = \frac{5}{2}x^{\frac{3}{2}}$$

2E. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2x-6}{\sqrt{x}}\right)$

$$\frac{d}{dx}\left(\frac{2x-6}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{2x}{x^{\frac{1}{2}}} - \frac{6}{x^{\frac{1}{2}}}\right)$$
$$= \frac{d}{dx}\left(2x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}\right)$$
$$= x^{-\frac{1}{2}} + 3x^{-\frac{3}{2}}$$

THE TWO TERMS IN THE NUMERATOR CAN BE WRITTEN AS SEPARATE FRACTIONS

REDUCING MISTAKES

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{BUT} \quad \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

1P. (a) $y = (4x^2 - 3x)(2x - x^3)$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(b) $\mathrm{g}(x) = 2\sqrt{x}\left(4x + x^2\right)$. Find $\mathrm{g}'(x)$.

2P. (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3x-2}{x}\right)$

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+4x^2}{2x}\right)$

3E. Find the gradient of the graph $y = 4x^3$ at the point where $x = 2$.

EVALUATE $\frac{dy}{dx}$ WHEN $x = 2$

$$\frac{dy}{dx} = 12x^2$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=2} = 12(2)^2 = 48$$

4E. Find the values of x for which the graph of $y = x^3 - 7x + 1$ has a gradient of 5.

$\frac{dy}{dx} = 5$

$$\frac{dy}{dx} = 3x^2 - 7$$

$$\therefore 5 = 3x^2 - 7$$

$$12 = 3x^2$$

$$4 = x^2$$

$$x = \pm 2$$

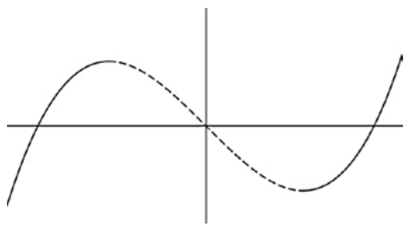
3P. Find the gradient of the graph $y = \frac{2}{x} + \sqrt{x}$ at the point where $x = 1$.

4P. Find the value of x for which the graph of $y = x^{\frac{4}{3}}$ has a gradient of $\frac{16}{15}$.

INCREASING AND DECREASING FUNCTIONS

- The sign of the gradient at a point tells you whether the function is increasing or decreasing at that point.
  - $\frac{dy}{dx} > 0 \Rightarrow$ The function is increasing (positive gradient)
  - $\frac{dy}{dx} < 0 \Rightarrow$ The function is decreasing (negative gradient)

INCREASING AND DECREASING FUNCTIONS



— $y = f(x)$ is increasing
--- $y = f(x)$ is decreasing

5E. (a) Find the range of values of x for which the function $f(x) = 2x^3 - 6x$ is decreasing.

$$f'(x) = 6x^2 - 6$$

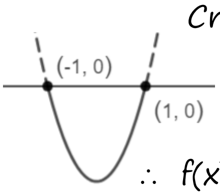
$f(x)$ is decreasing when $f'(x) < 0$

$$\therefore 6x^2 - 6 < 0$$

Critical values at $6x^2 - 6 = 0$

$$x^2 = 1$$

$$x = \pm 1$$



$\therefore$ f(x) is decreasing when $-1 < x < 1$

5E. (b) Show that the function $3x^3 + 5x$ is increasing for all values of x.

$$f'(x) = 9x^2 + 5$$

Since $x^2 \geq 0$, $f'(x) > 0$ for all x.

$\therefore$ $f(x)$ is always increasing.

3P. Find the range of values of x for which $y = 4x^2 + \frac{1}{x}$ is increasing.